Unstructured Orthogonal Meshes for Modeling Coastal and Ocean Flows

Olga Kleptsova

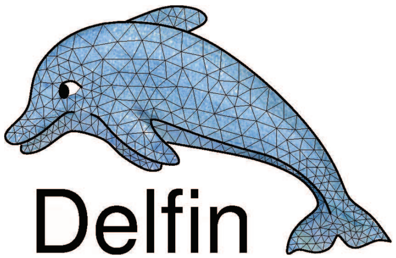
Delft University of Technology

Outline

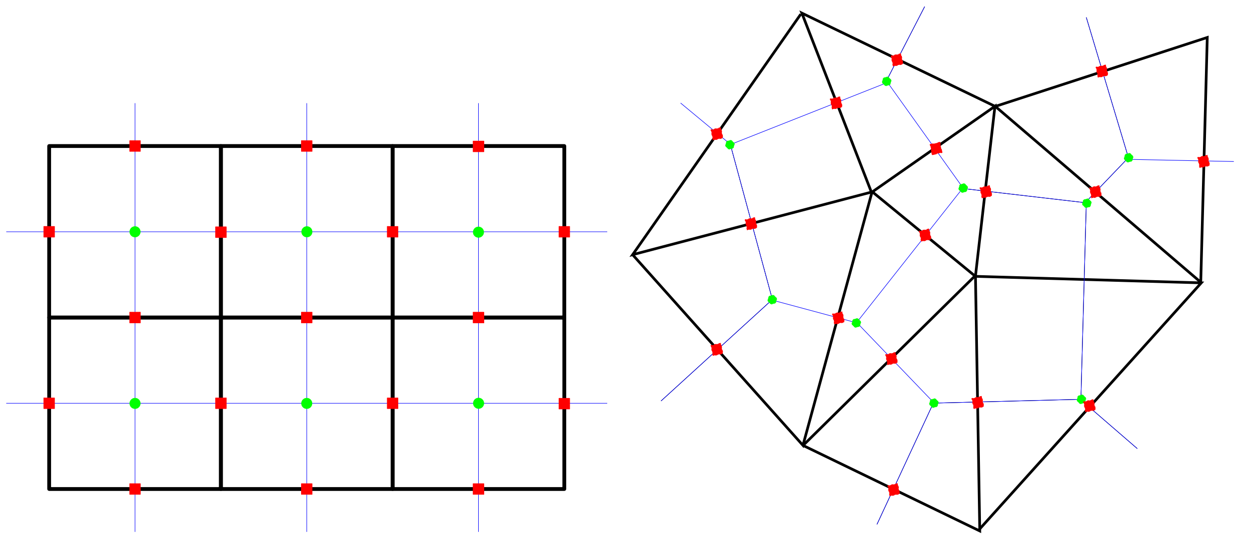
- Summary of the numerical method
- North Sea tidal model
- Indian Ocean Tsunami simulation

Delfin: scheme properties

- Orthogonal unstructured C-grid
- Geopotential $z-$ layer coordinates
- Casulli and Walters (2000):
  - Semi-implicit finite volume method
  - Advection and Coriolis terms are treated explicitly
- Eulerian advection



C-grid discretisations



- Only normal components of velocity are solved for
- Tangental velocity components are interpolated
- Interpolation may introduce accuracy and stability problems (Espelid et al, 2000)

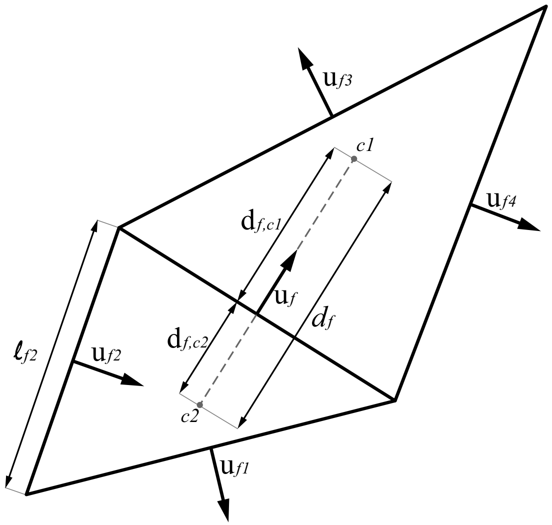
Tangential velocity interpolation

$$\frac{d}{dt}\begin{bmatrix} \mathbf{u} \\ \eta \end{bmatrix} = \begin{bmatrix} F & P \\ C & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ \eta \end{bmatrix}$$

- Propagation matrix of semi-discrete system should only have imaginary eigenvalues (Espelid et al, 2000).
  - Skew-symmetric matrix ($A^T = -A$)
  - Similar to skew-symmetric ($D^{-1}AD$ is skew-symmetric.)

Tangential velocity interpolation

- Perot (2000): Velocity reconstruction for $2D$ Navier-Stokes equations:
  - Cell velocity vector: $A_c\mathbf{u}_c = \sum_f s_{f,c} l_f u_f \mathbf{r}_f$
  - Face velocity vector: $\mathbf{u}_f = \sum_c \delta_{f,c}\frac{d_{f,c}}{d_f}\mathbf{u}_c$



where

- $s_{f,c} = (\mathbf{n}_f \cdot \mathbf{N}_f)$,
- A_c - face area,
- $\mathbf{r}_f = s_{f,c} d_{f,c} \mathbf{n}_f$ - position vector pointing from cell to face centre

Tangential velocity interpolation

- Perot (2000): No energy conservation in the presence of varying bottom topograthy
- Ham et al (2004): $\mathbf{u}_f = \sum_c \delta_{f,c} \frac{h_c}{h_f} \frac{d_{f,c}}{d_f} \mathbf{u}_c$
- Espelid et al (2000,2004*):

$$A_c \mathbf{u}_c = \sum_f s_{f,c} l_f \frac{\sqrt{h_f}}{\sqrt{h_c}} u_f \mathbf{r}_f, \;\; \mathbf{u}_f = \sum_c \delta_{f,c} \frac{d_{f,c}}{d_f} \frac{\sqrt{h_c}}{\sqrt{h_f}} \mathbf{u}_c$$

- Kleptsova et al (2009): Coriolis tilting test case shows growth of energy using the above

Tangential velocity interpolation

- Perot's reconstruction is based on the identity

$$\int_{\Omega} \omega dV + \int_{\Omega} \mathbf{r}(\nabla \cdot \omega) dV = \int_{\partial\Omega_f} (\omega \cdot \mathbf{N}) \mathbf{r} dA$$

  It is valid for any
  - polygonal volume Ω with piecewise smooth boundary $\partial\Omega$
  - continuously differentiable vector field ω
  - position vector $\mathbf{r} = \mathbf{x} - \mathbf{x_0}$ with an arbitrary origin $\mathbf{x_0}$.
- (2)$\equiv 0$ for $\mathbf{u} = [u, v, w]$, but not for $\bar{\mathbf{u}} = \frac{1}{h}\int_b^\eta \mathbf{u} dz$

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Tangential velocity interpolation

- Use the identity with $\omega = \mathbf{u}$

$$\int\limits_{\Omega} \mathbf{u} dV = \int\limits_{\partial\Omega_f} (\mathbf{u} \cdot \mathbf{N}) \mathbf{r} dA$$

- Integrate over a (prismatic) cell/water column

$$h_c A_c \mathbf{u}_c = \sum_f s_{f,c} h_f l_f u_f \mathbf{r}_f + A_c \left(w_t \mathbf{r}_t - w_b \mathbf{r}_b \right)$$

  - Where $\mathbf{r}_t, \mathbf{r}_b -$ position vectors pointing from the cell center to the centers of the top and bottom faces

- Note: If $\mathbf{r}_t, \mathbf{r}_b$ are not strictly vertical, w_b, w_b may contribute to $\mathbf{u}_c^{xy}$. In the case of w interpolated from the continuity equation, this may make the matrix not skew-symmetric.

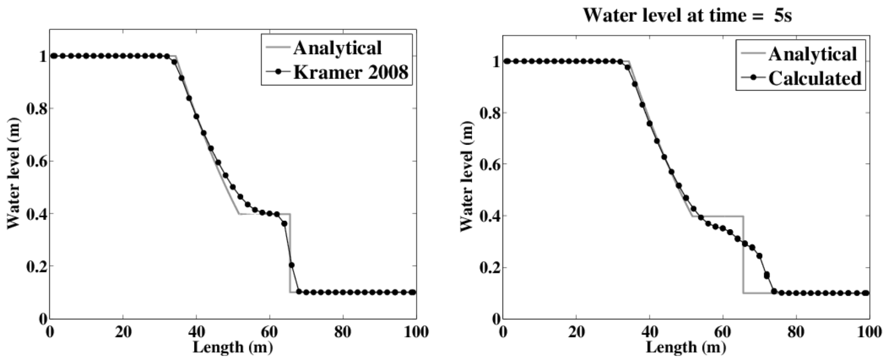
Advection discretisation

- We use an Eulerian advection scheme by Kleptsova et al (2010)
- Multi-layer variant of scheme by Kramer and Stelling (2008)
- Momentum conservative
- Time step limitations

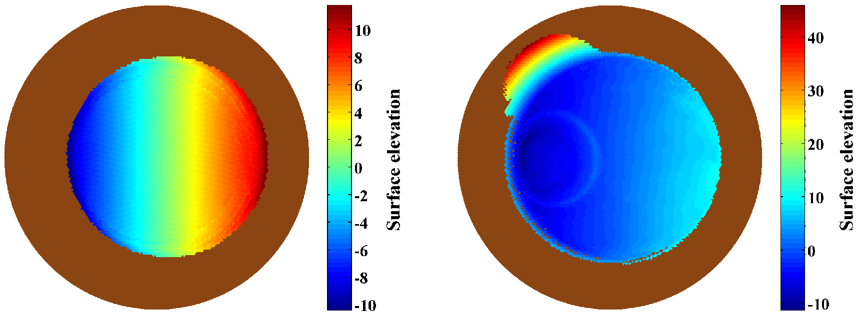
$Z-$layer discretization

- Kleptsova et al (2010):

  $z-$layer + C-grid $\Rightarrow$ Accuracy problem in advection dominated flows



- The same holds for Coriolis dominated flows

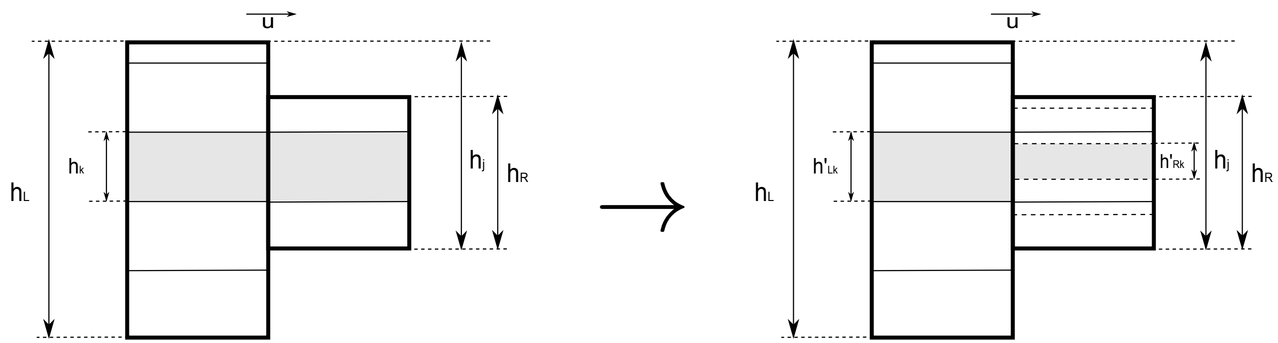


$Z-$layer discretization

- Stepwise discontinuous representation of the topography and free surface
- Variable thickness of the bottom layer
- Vanishing top layer
- $\frac{h_{f,i}}{h_{c,i}} \neq \frac{h_{f,b}}{h_{c,b}}$

Kleptsova et al (2010): in absence of bottom friction

- Momentum equation should be identical for all of the layers
- Column to columside water depth ratio should be the same as cell to face height ratio



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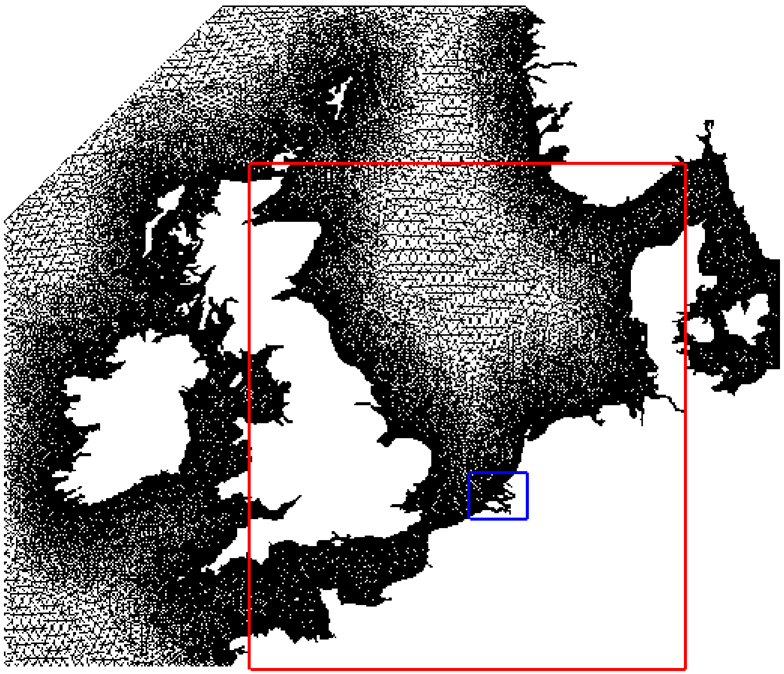
North Sea tidal model



- Boundaries are coincident with DCSM98
- Bathymentry is that of DCSM98
- Coarse grid:
  - 131 thousand cells
  - 1-20 km resolution
  - Orthogonal variant of Ham(2006) grid
- Fine grid:
  - 690 thousand cells
  - 10m-20km resolution

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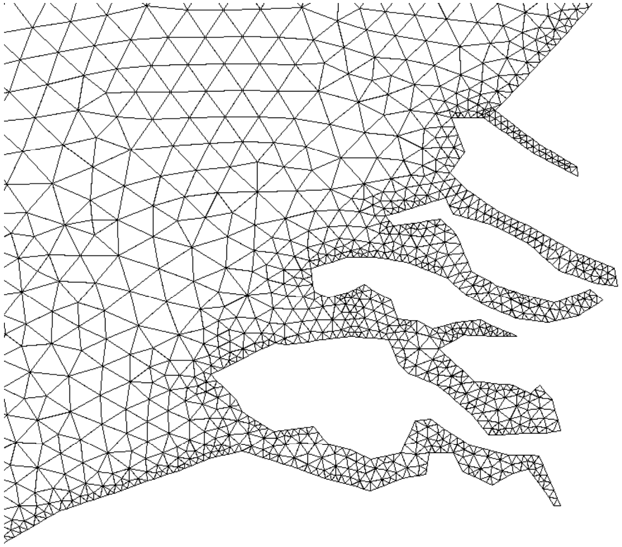
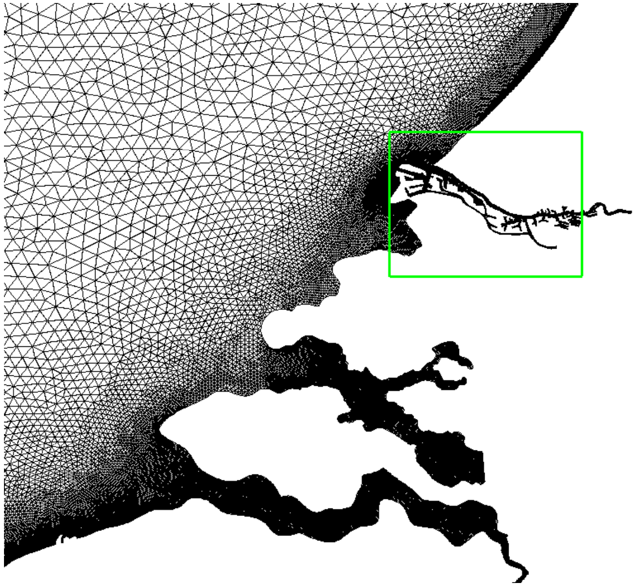
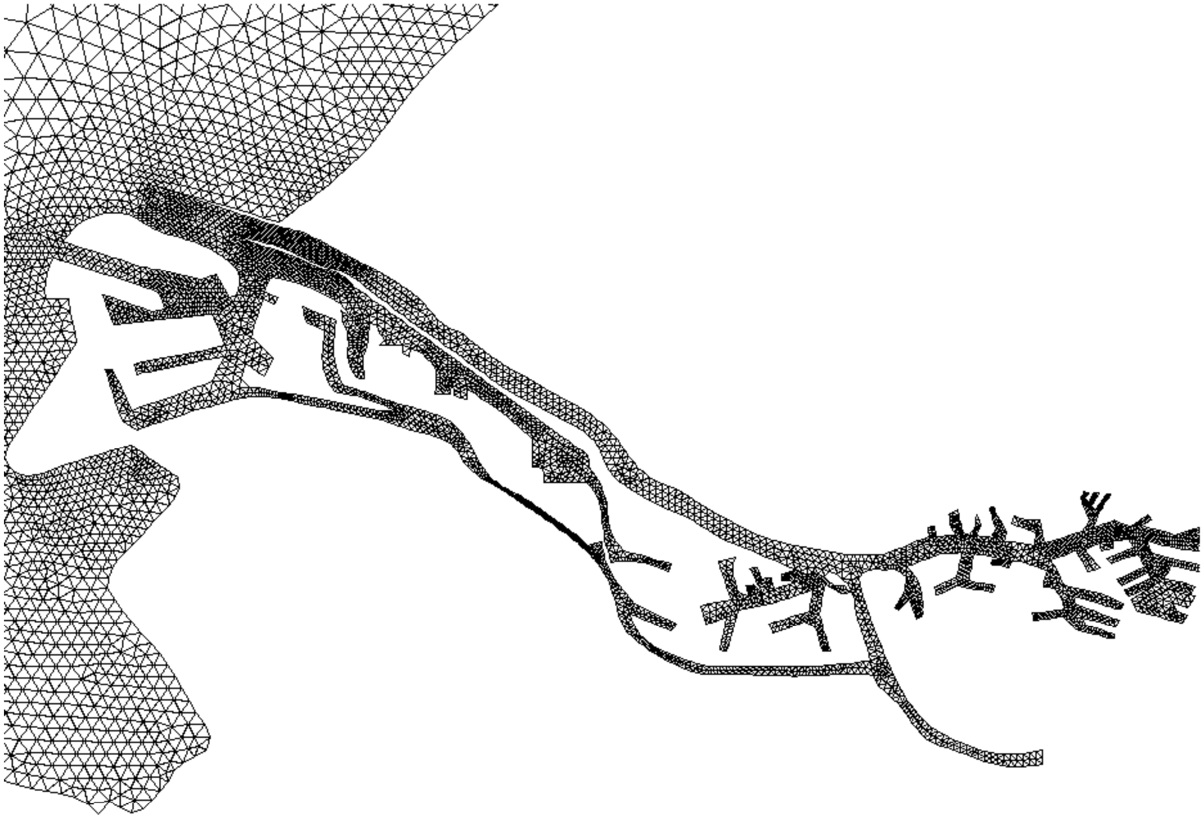
North Sea tidal model



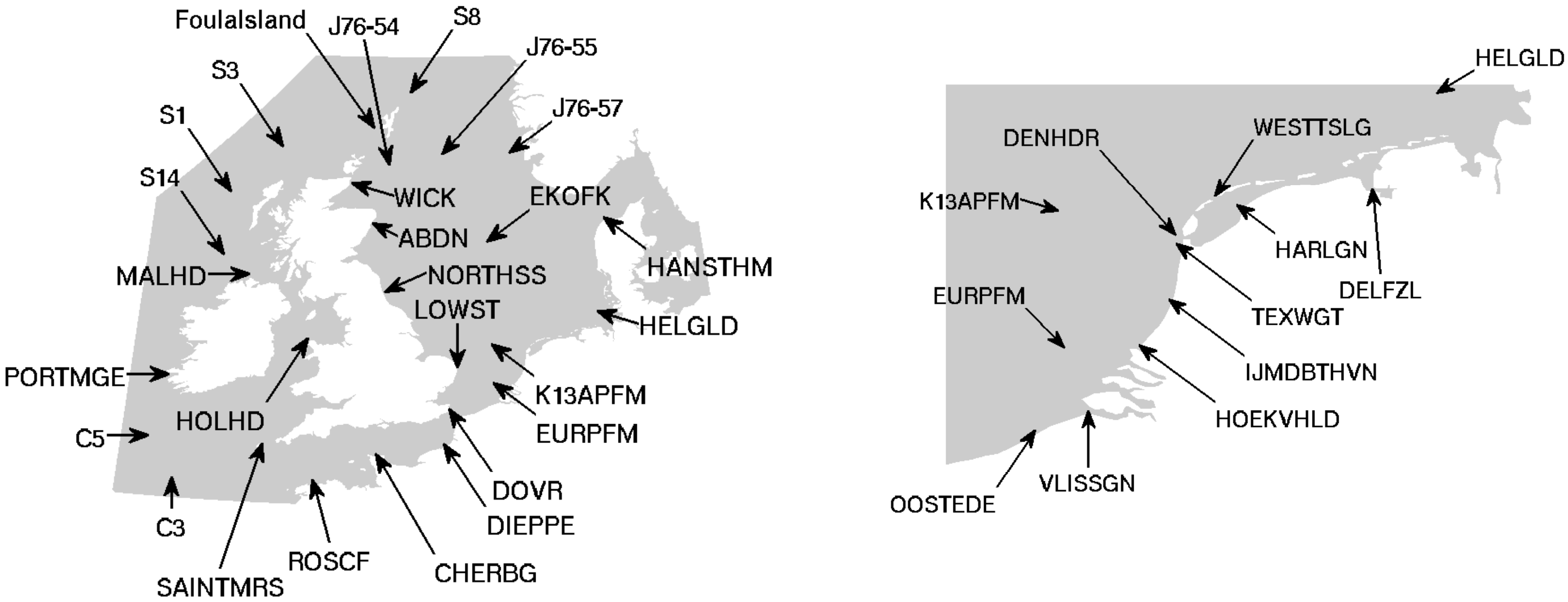


North Sea tidal model

Coastlines are provided by Gerard Dam, Svasek Hydraulics



North Sea tidal model



- 27 tidal points where spectrum is specified
- Only diurnal lunar (M2) component is considered
- Data for 36 stations were available for comparison

North Sea tidal model

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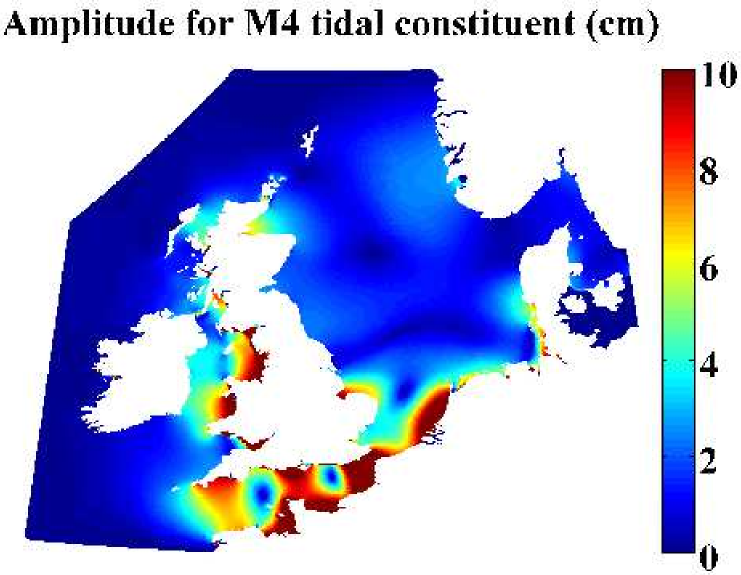
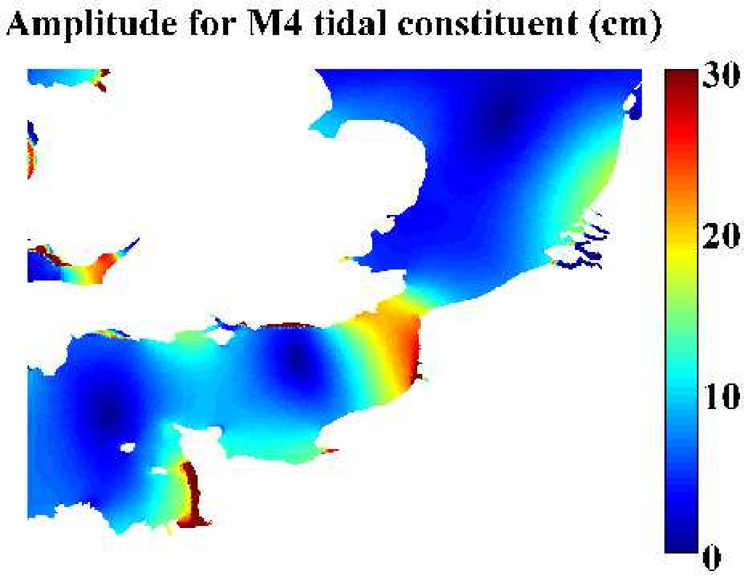
- Latitude $55°$
- Timestep 5min/10min for fine/coarse grid
- Linear dynamics
- Amplitude error < 20cm for 72% (26 of 36) stations
- Phase error $<$ $10°$ for 58% (21 of 36) stations

North Sea tidal model

- Model forced with:
  - semidiurnal tides M2, S2, K2 and N2,
  - diurnal tides K1, O1, P1, Q1
- The shallow water tides M4, M6 were generated on the shelf





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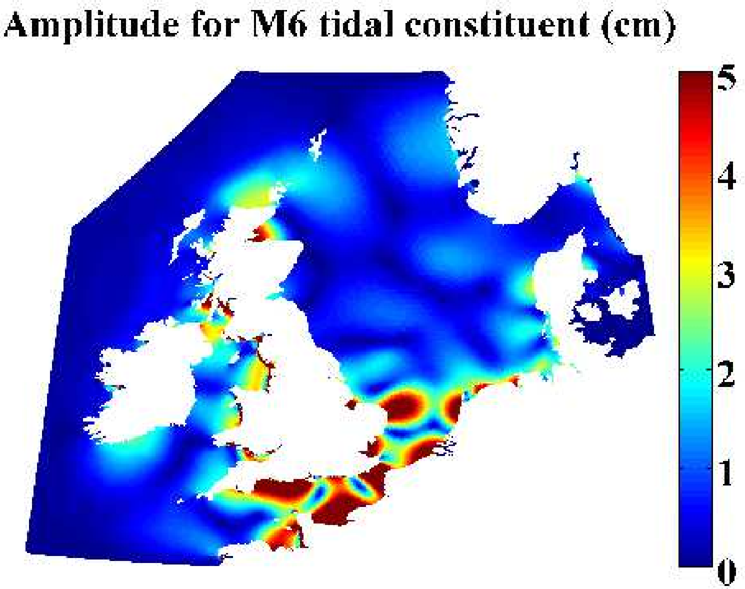
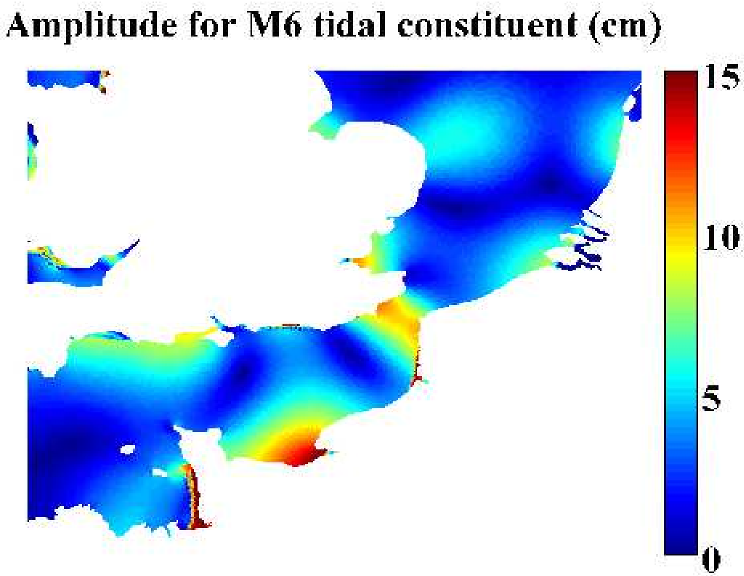
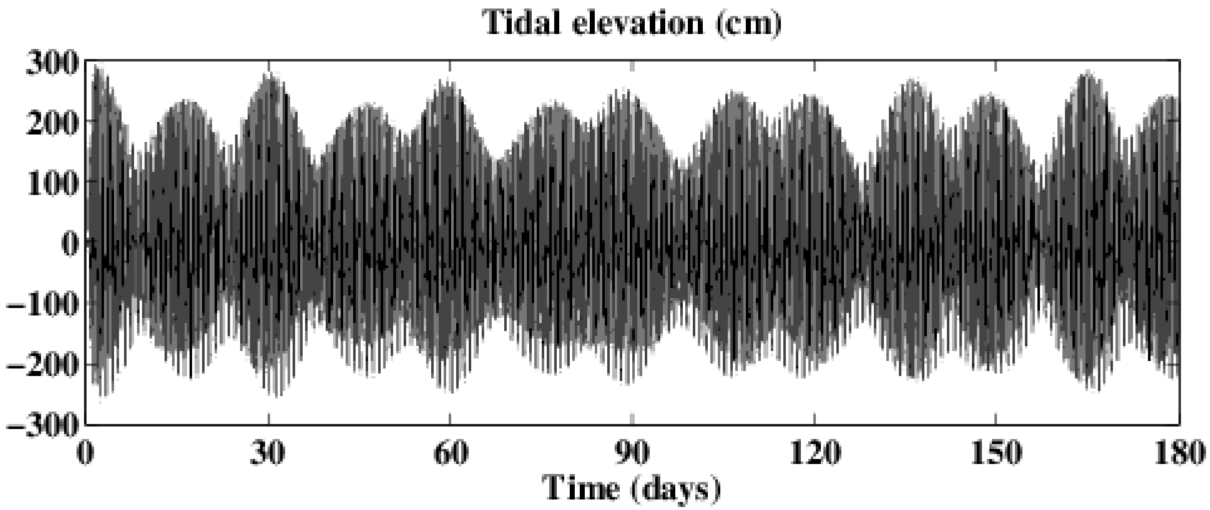


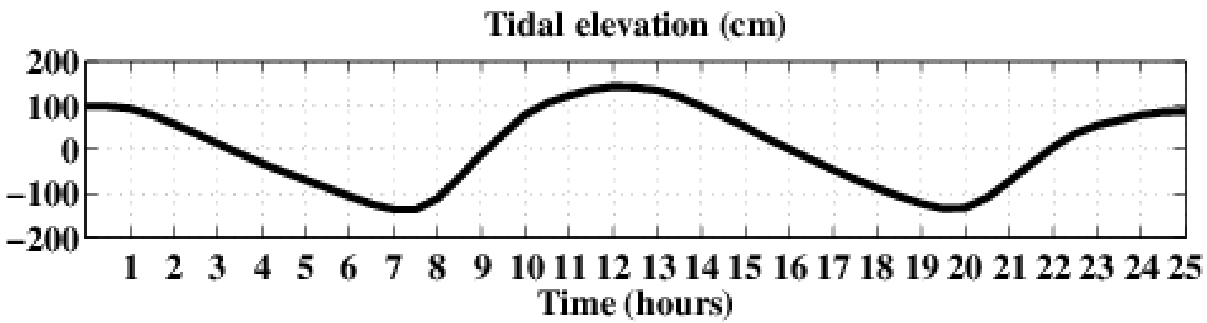


North Sea tidal model

- Spring-neap variation of the water level at the Harlingen station

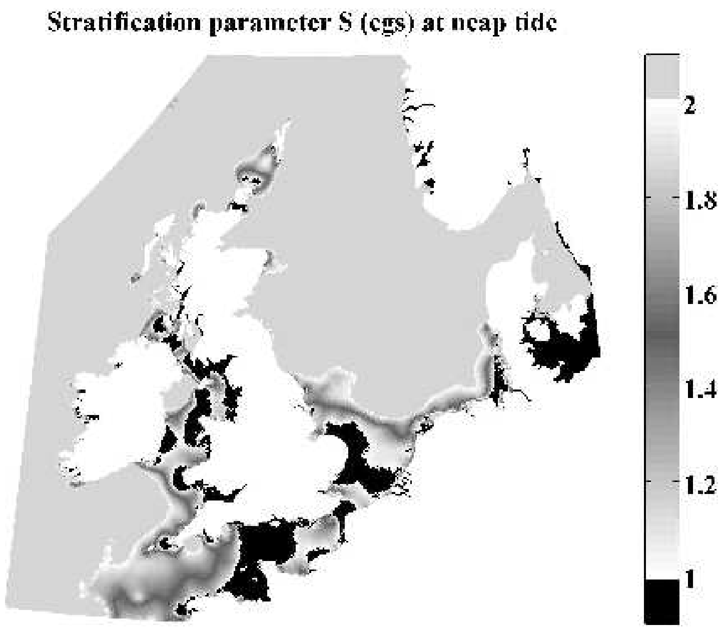


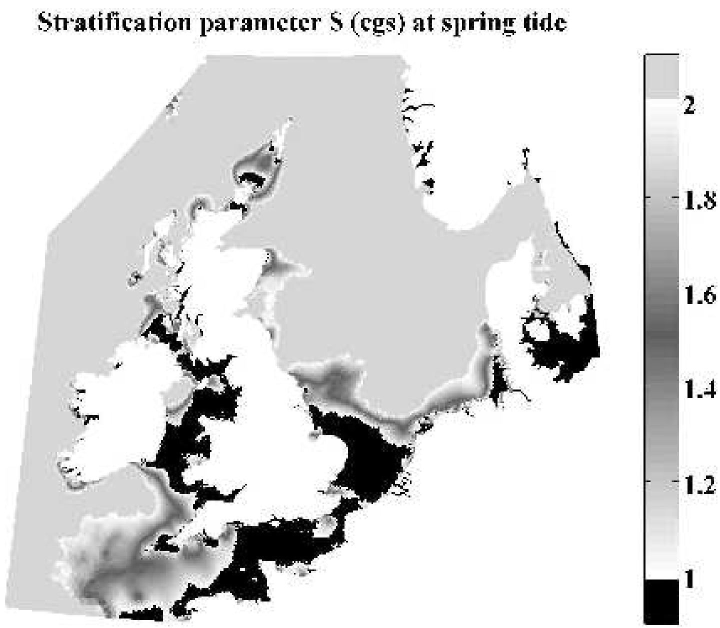
- Flood-ebb asymmetry at Harlingen station



North Sea tidal model

- B. Sinha and R. Pingree (1978): Stratification parameter $S = \log_{10}\left(\frac{h}{C_d|u_s|^3 10^4}\right)$
  - $C_d = 0.0025$
  - $S > 2-$ stratified, $S < 1-$ well-mixed





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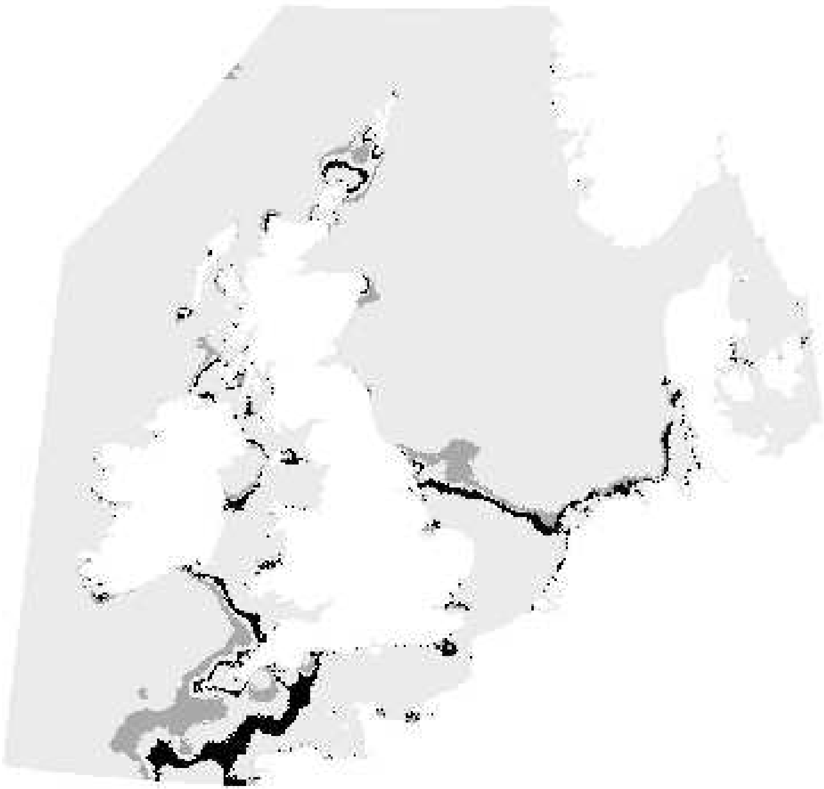
North Sea tidal model



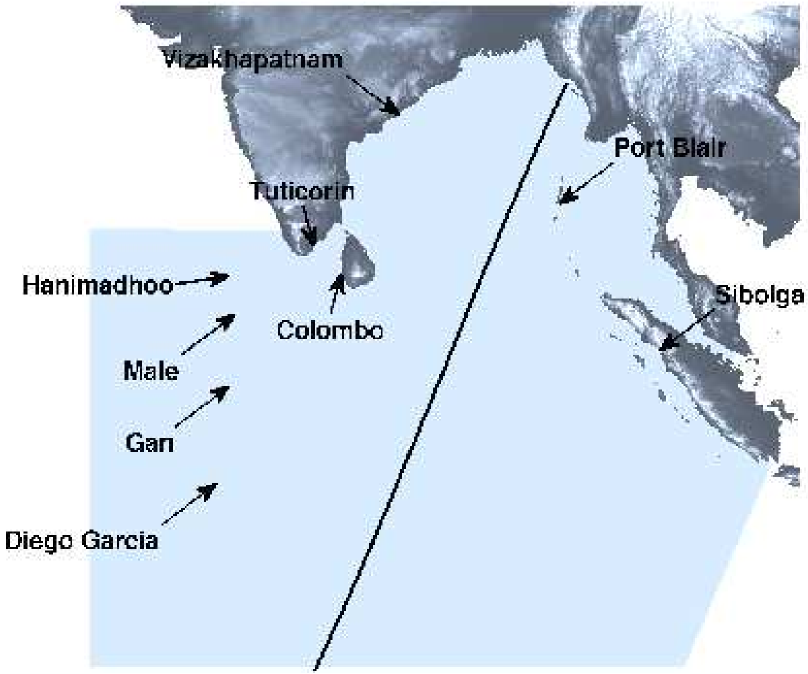
Black: neap tide

Gray: spring tide

Indian Ocean Tsunami

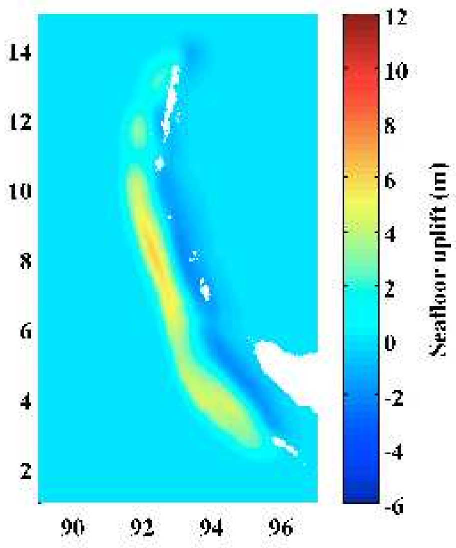
- Major difficulty lies in determining of a precise fault mechanism
- A number studies has been done using GPS and seismic data
- Co-seismic displacement data are validated using the results of tsunami models
- Results are compared to a number of independent data sources

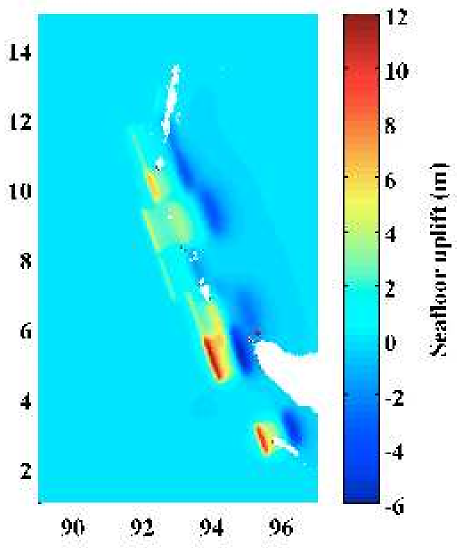
Indian Ocean Tsunami



Indian Ocean Tsunami

- Model 1: GPS invertion, Hoechner et al. (2008)
  - Rupture velocity $3.7km/s$ for the first $200km$, when $2km/s$
  - Total rupture time $10min$
- Model 2: Coastal coseismic vertical deformations and wave forms at tide gauges, Tanioka et al. (2006)
  - Rupture velocity $1.7km/s$
  - Total rupture time $12min$





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Indian Ocean Tsunami

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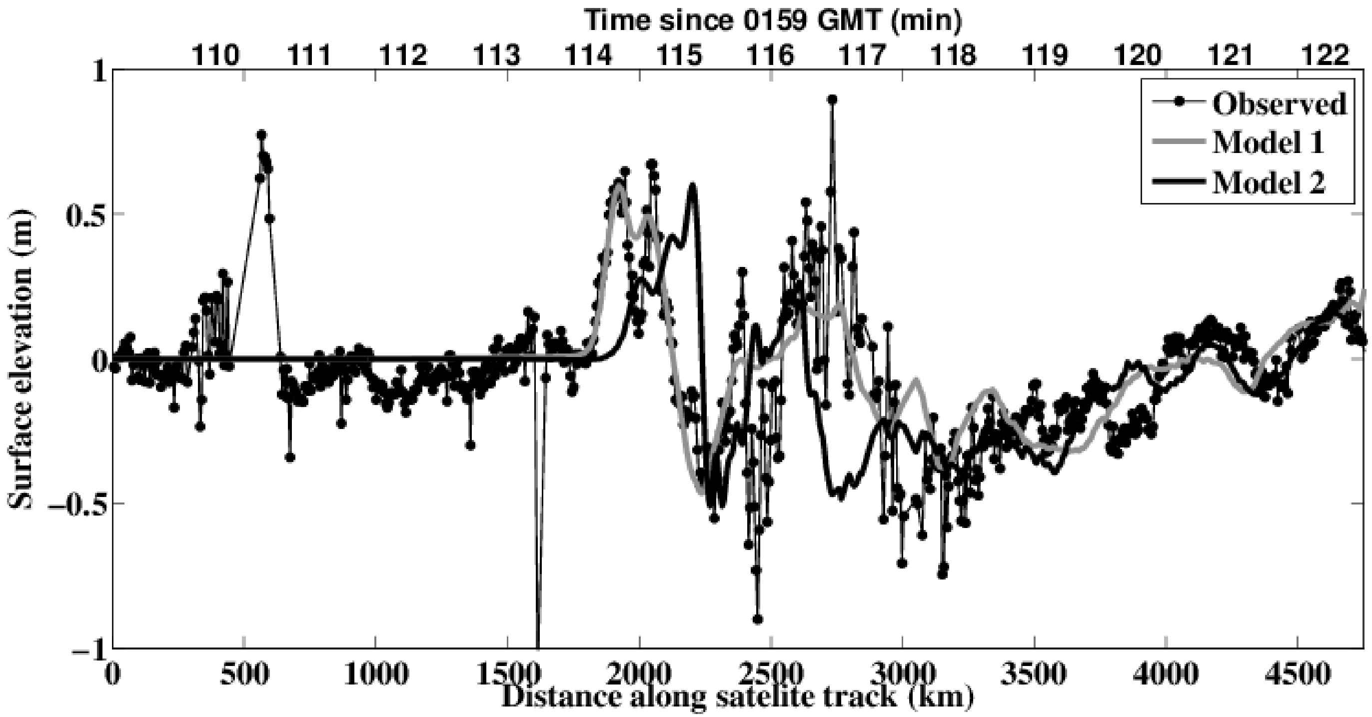
Indian Ocean Tsunami

Site	Coordinates	Sampling interval of tide gauge (min)	Arrival times of the leading incident tsunami waves (in minutes since the earthquake started)		
			Tide gauge	Model 1 error	Model 2 error
Sibolga, Indonesia	$01.75°$ N; $98.75°$ E	3	107	-24	-27
Tuticorin, India	$08.75°$ N; $78.20°$ E	6	205	8	15
Vizakhapatnam, India	$17.65°$ N; $83.28°$ E	5	156	-15	-11
Colombo, Sri Lanka	$06.93°$ N; $79.83°$ E	2	170	-9	-2
Male, Maldives	$04.18°$ N; $73.52°$ E	4	195	-5	1
Diego Garcia, UK	$07.30°$ S; $72.38°$ E	6	226	-16	-8
Hanimadhoo, Maldives	$06.77°$ N; $73.18°$ E	2	211	-15	-9
Gan, Maldives	$00.68°$ S; $73.17°$ E	4	197	-7	3
Port Blair, India	$11.68°$ N; $92.77°$ E	2	15	8	-

Indian Ocean Tsunami



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Indian Ocean Tsunami

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Indian Ocean Tsunami

- Two initial sea surface displacement fields were compared
- There are still uncertainties in the source description
- Satellite altimetry, inundation measurements, arrrival data are valuable data sources
- It is not enougth to compare only to one data source

Questions?