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Morphodynamic modeling on unstructured grid: integrating a Weighted Essentially Non-Oscillatory scheme and a multi grain-size approach.

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Sandy coastal environments subjected to tide and waves action are interesting for several reasons:

- fast morphological evolutions,
- many physical processes and their interaction,
- variable space and time scales,
- concentration of environmental, social and economic issues,
- singularity of each studied site,…

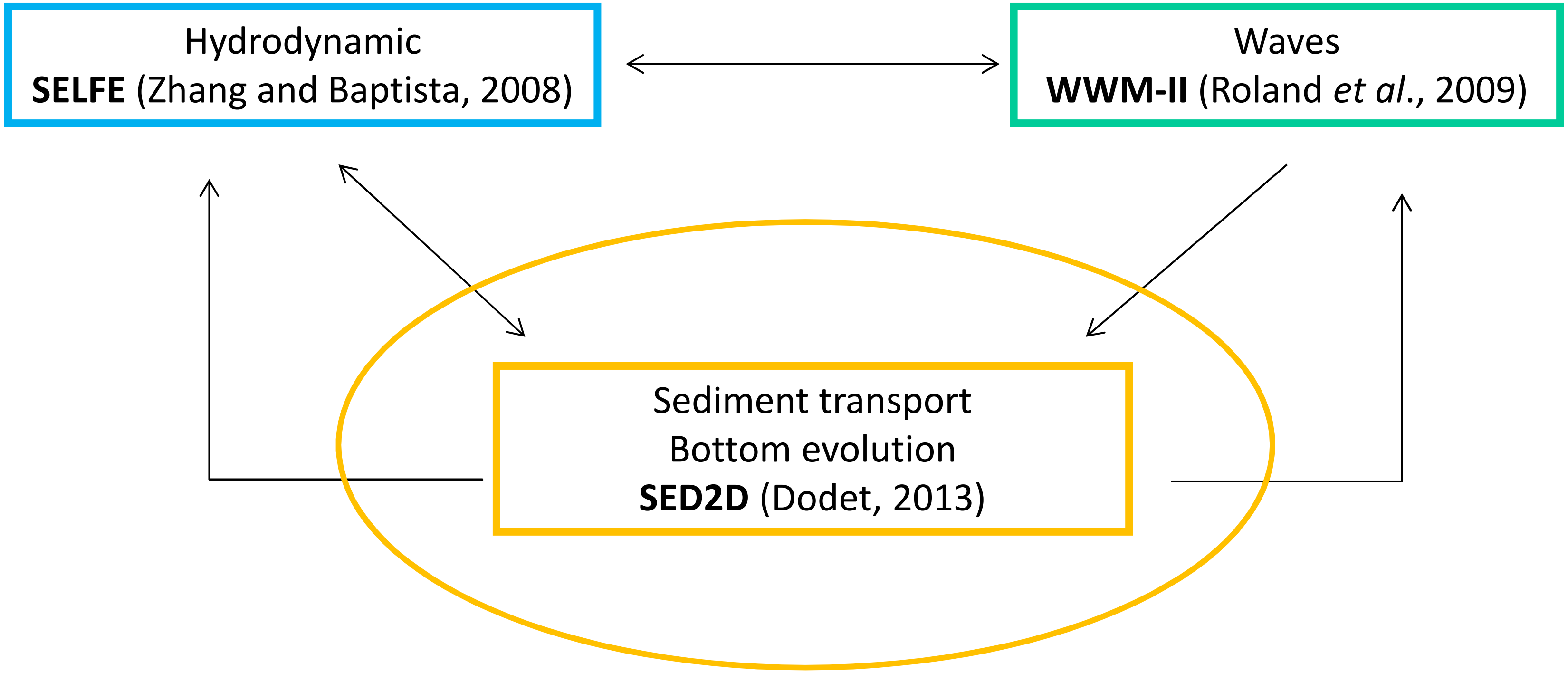
Morphodynamic modeling systems are valuable tools for studying evolution of sandy coastal environnements.

⇒ In our 2DH case, two specific improvements are presented in response to common limitations of morphodynamic models:

1) Integration of an oscillations-free numerical scheme (oscillations being due to the non-linear coupling between the sediment transport module and the bed evolution module)

2) To consider heterogeneous sediment rather than homogeneous sediment

Modeling system:

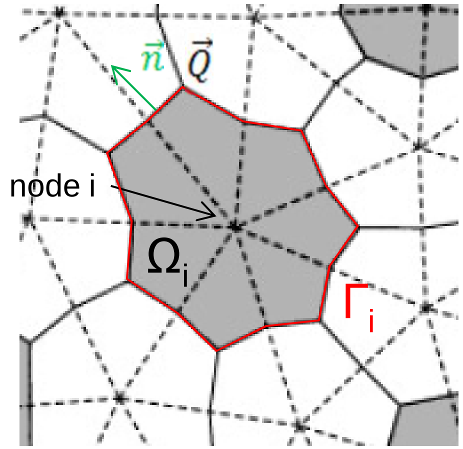


Collaborative work:

- Virginia Institute of Marine Science
- Oregon Health and Science University
- National Laboratory for Civil Engineering - Lisbon
- Technical University of Darmstadt
- UMR 7266 LIENSs - La Rochelle
- …

The sediment continuity / Exner equation is solved by the bottom evolution module to compute the bed change at each grid node:

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-\lambda}\nabla_{\mathbf{x}} Q = 0$$



Node-centered control volume

- $\mathbf{x} = (x,y)$
- bed level elevation: $z_b(\mathbf{x},t)$
- sediment porosity: λ
- depth-integrated sediment transport rate ($m^3.s^{-1}.m^{-1}$) computed at element centres: $Q=(Q_x,Q_y)$

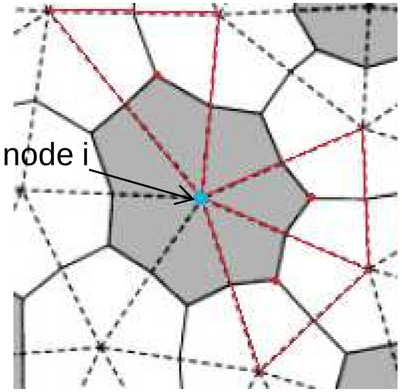
$\Rightarrow$ semi-discrete finite volume formulation of Exner equation: $$\frac{d}{dt}\int_{\Omega_i} z_b\, d\Omega = -\frac{1}{1-\lambda}\int_{\Gamma_i} Q.n\, d\Gamma$$

$\Rightarrow$ fully-discrete finite volume form (Euler explicit temporal discretization): $$\Delta z_{b,i} = -\frac{\Delta t}{|\Omega_i|(1-\lambda)}\int_{\Gamma_i} Q.n\, d\Gamma$$

- bed change over the morphological time step Δt: $\Delta z_{b,i}$
- control volume for node i: Ω_i
- boundary of Ω_i: Γ_i
- outward unit normal to Γ_i: n

A reconstruction polynomial $P_i(\mathbf{x})=(P_{x,i}(\mathbf{x}),P_{y,i}(\mathbf{x}))$ is determined for each control volume Ω_i to interpolate sediment fluxes at Γ_i, instead of considering constant sediment flux within each element.

$\Rightarrow$ $P_i(\mathbf{x})$ is computed as a weighted mean of linear polynomials: $P_i(\boldsymbol{x}) = \sum_{k=1}^{N} \omega_k p_k(\boldsymbol{x})$



Each linear polynomial is computed from a stencil composed of 3 elements neighboring node i:

$$p_{x,m}(\boldsymbol{x}) = a_{x,m}x + b_{x,m}y + c_{x,m} \qquad p_{y,m}(\boldsymbol{x}) = a_{y,m}x + b_{y,m}y + c_{y,m}$$

such as

$$p_{x,m}(\boldsymbol{x}_c) = Q_x \qquad p_{y,m}(\boldsymbol{x}_c) = Q_y$$

Note: all combinations (N_i) of 3 neighboring elements are considered for each control volume but a fixed number $N<N_i$ is used to compute $P_i(\mathbf{x})$.

The corresponding weight for each linear polynomial is computed following Friedrich (1998) as:

$$\omega_m = \frac{(\epsilon + OI_m)^{-r}}{\sum_{k=1}^{N}(\epsilon + OI_k)^{-r}}$$

With OI_m being an oscillation indicator for each stencil, aiming to measure the smoothness of $p_m(\mathbf{x})$:

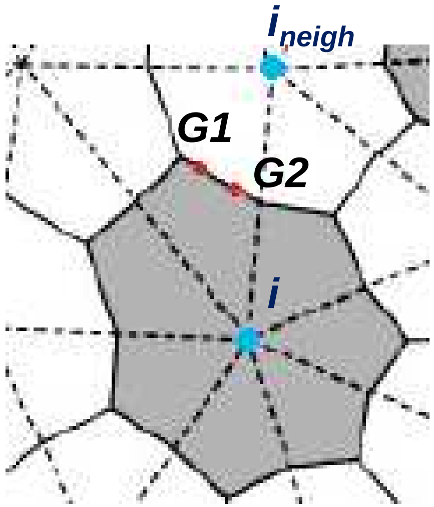
$$OI_m = OI_{x,m} + OI_{y,m} \qquad OI_{x,m} = \left[\int_{\Omega_i} dX^{-2}\left[\left(\frac{\partial p_{x,m}(x,y)}{\partial x}\right)^2 + \left(\frac{\partial p_{x,m}(x,y)}{\partial y}\right)^2\right]d\Omega\right]^{1/2} \qquad OI_{x,m} = \sqrt{\frac{|\Omega_i|}{dX^2}\left(a_{x,m}^2 + b_{x,m}^2\right)}$$

Back to the fully-discrete finite volume form: $\Delta z_{b,i} = -\frac{\Delta t}{|\Omega_i|(1-\lambda)}\int_{\Gamma_i} Q.n\, d\Gamma$

$\Rightarrow$ Sediment flux Q(**x**) is replaced by P_i(**x**): $\int_{\Gamma_i} Q.n\, d\Gamma = \int_{\Gamma_i} P_i.n\, d\Gamma = \sum_j \int_{\Gamma_{i,j}} P_i.n\, d\Gamma$

$\Rightarrow$ A 2 points Gaussian integration formula is used to discretize each line integral:

$$\int_{\Gamma_{i,j}} P_i.n\, d\Gamma \approx |\Gamma_{i,j}|\frac{1}{2}\left(P_i(G_1) + P_i(G_2)\right).n = F_{i,j}$$



2 points Gaussian integration

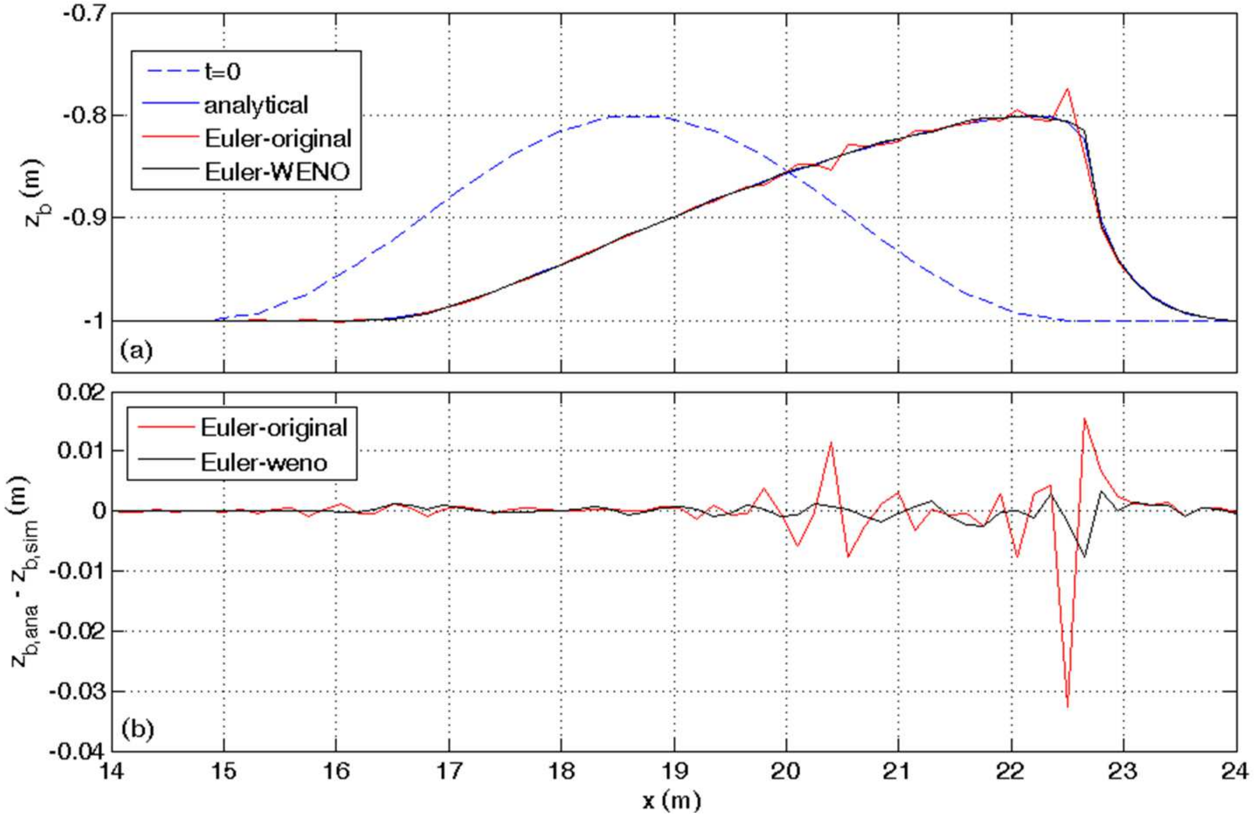
Finally the numerical flux of Godunov is used to compute the final flux at each $\Gamma_{i,j}$:

$$F_{i,j}^{final} = \begin{cases} \max\left(F_{i,j}, F_{i_{neigh},j}\right) & if\ z_b(i) \geq z_b(i_{neigh}) \\ \min\left(F_{i,j}, F_{i_{neigh},j}\right) & if\ z_b(i) < z_b(i_{neigh}) \end{cases}$$

Note: monotonicity of the flux is assured since $F_{i,j}^{final} \in [F_{i,j}, F_{i_{neigh},j}]$

Test case 1: Migrating sandwave under unidirectional and stationary flow in a straight channel

Comparison between numerical result and analytical solution using simple transport rate function $Q=aU^b$; $\Delta t = 2$ s and $\Delta x = 0.15$ m, leading to a maximum Courant number of ~0.1.

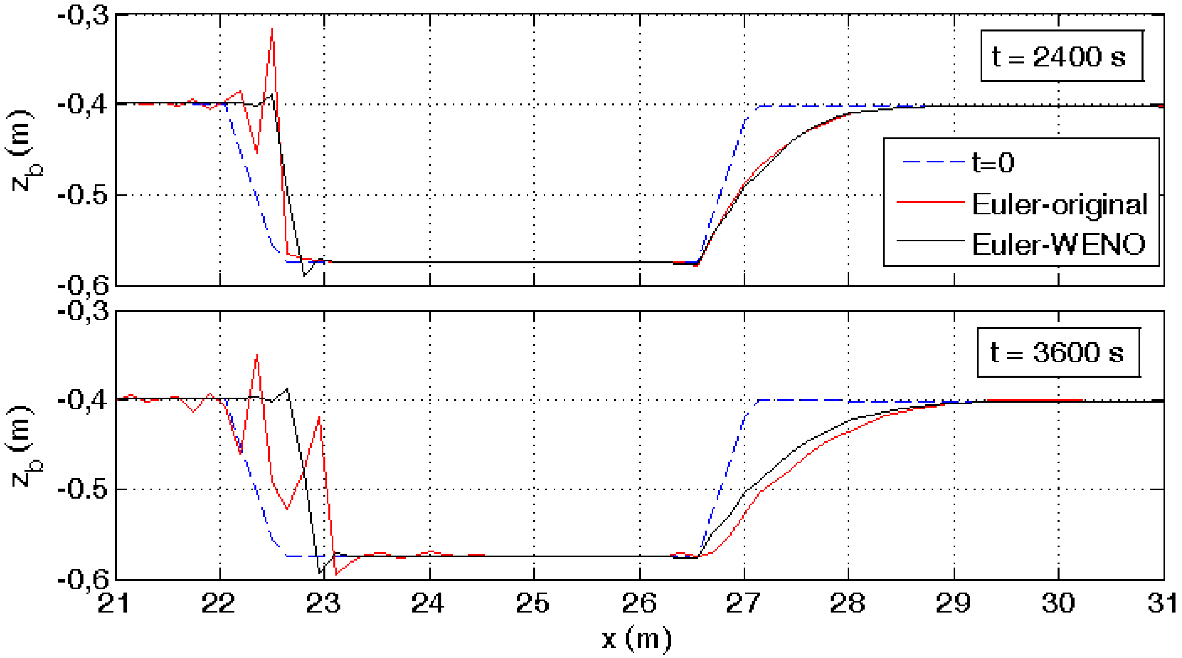


Comparison of Euler-WENO scheme and Euler-original scheme results to analytical solution at y = 0.75 m, without any filtering or artificial diffusion. (a) Bed profiles at t = 500 s ; (b) Difference with analytical solution.

⇒ Enhanced accuracy with the WENO scheme compared to the original scheme.

Test case 2: Migrating trench under unidirectional and stationary flow in a straight channel

Based on a laboratory experiment of Van Rijn (1987), this test case allows us to test the robustness of the numerical scheme in response to strong initial bed level gradients.



Comparison of original and WENO schemes results for the trench migration test case (y = 0.75 m).

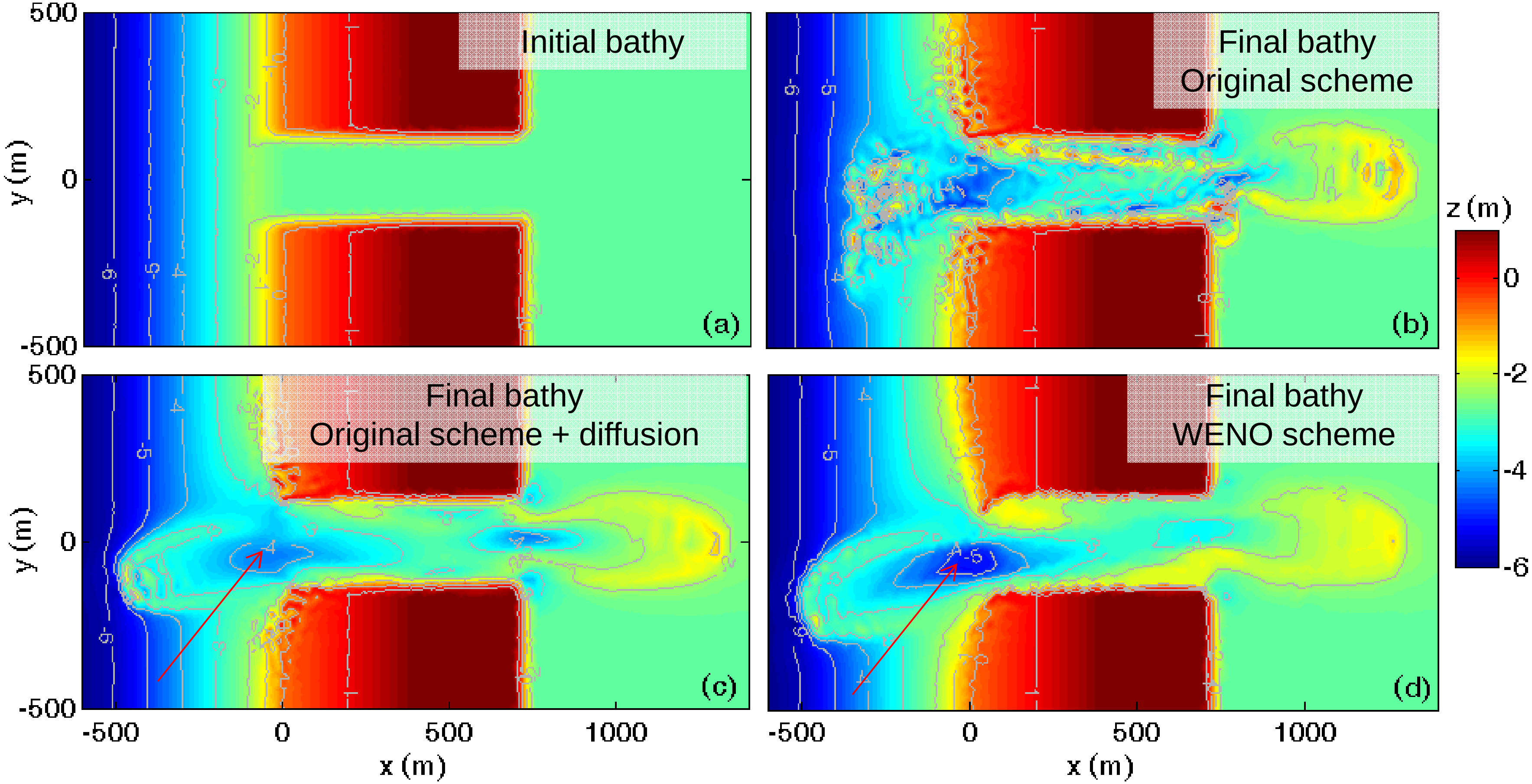
- bed-load and suspended load computed with Van Rijn (2007)
- initial flow velocity: 0.49 $m.s^{-1}$
- median diameter: 0.14 mm
- Δt = 1 s

$\Rightarrow$ Enhanced stability of the WENO scheme compared to the original scheme

Note: the amplitude of bed level perturbations for the WENO scheme (which could be thought as emerging oscillations) does not increase in time, verifying the ability of the scheme to capture shocks.

Test case 3 : Idealized inlet

Parameters: M2 tide ; waves with H_s = 1 m, T_p = 10 s ; transport formula of Van Rijn (2007) ; d50 = 0.5 mm ; Δt = 30 s.



(a) Initial bathymetry. 30-days simulated bathymetry for (b) Euler-original scheme ; (c) Euler-original scheme with diffusion ε=3 ; (d) Euler-WENO scheme.

⇒ Cleaner and more realistic bathymetry obtained with the WENO scheme

Using a WENO scheme in our 2DH morphodynamic model, rather than the original scheme, presents several improvements:

- higher accuracy (sandwave migration test case)

- higher stability and shock capturing behaviour (trench migration test case)

- realistic evolution of an idealized inlet submitted to tide and waves action

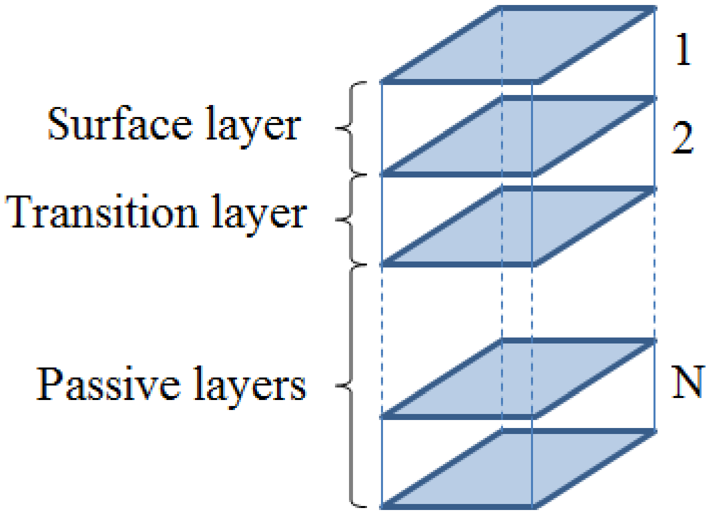
$\Rightarrow$ This scheme is preferred to the common additional diffusion method and/or the bathymetric filtering method because it is working independently of the bed level gradients (no over-smoothing effect like with diffusion).

$\Rightarrow$ Alternative to the DG scheme of Kubatko *et al*. (2006) ; comparisons between both schemes would be very interesting.

Perspective: testing more elaborated temporal discretization for WENO scheme

Integration of a multi grain-size approach is done following a multi-class and multi-layer method based on Reniers *et al*. (2013):

Each sediment class j is defined by a specific median grain diameter ($d50_j$) and a corresponding fraction (F_j).



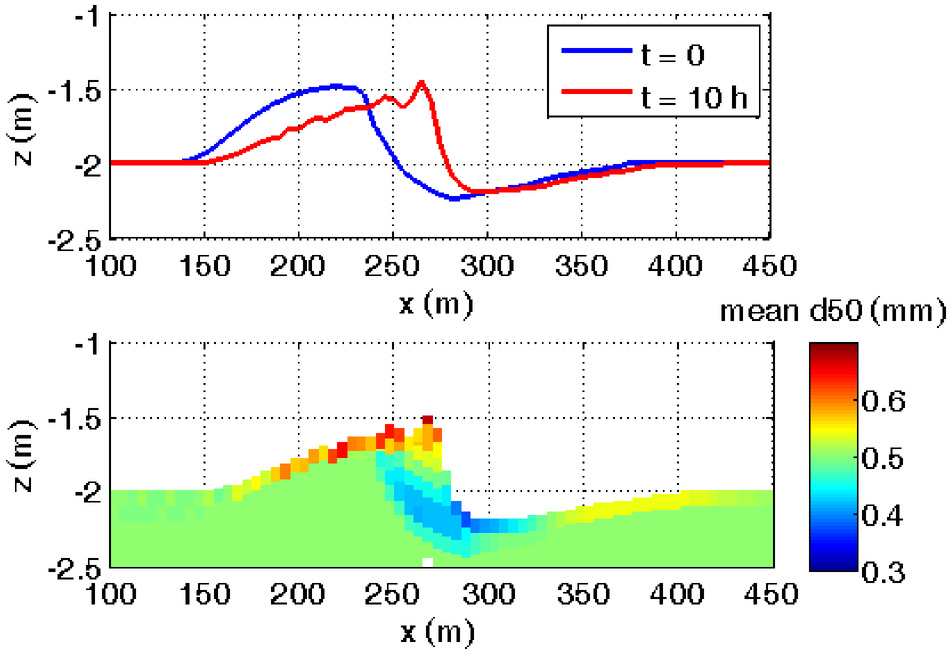
Each sediment layer *L* is defined by a specific thickness, which is variable only for the transition layer.

=> The mean d50 in each layer is computed as:

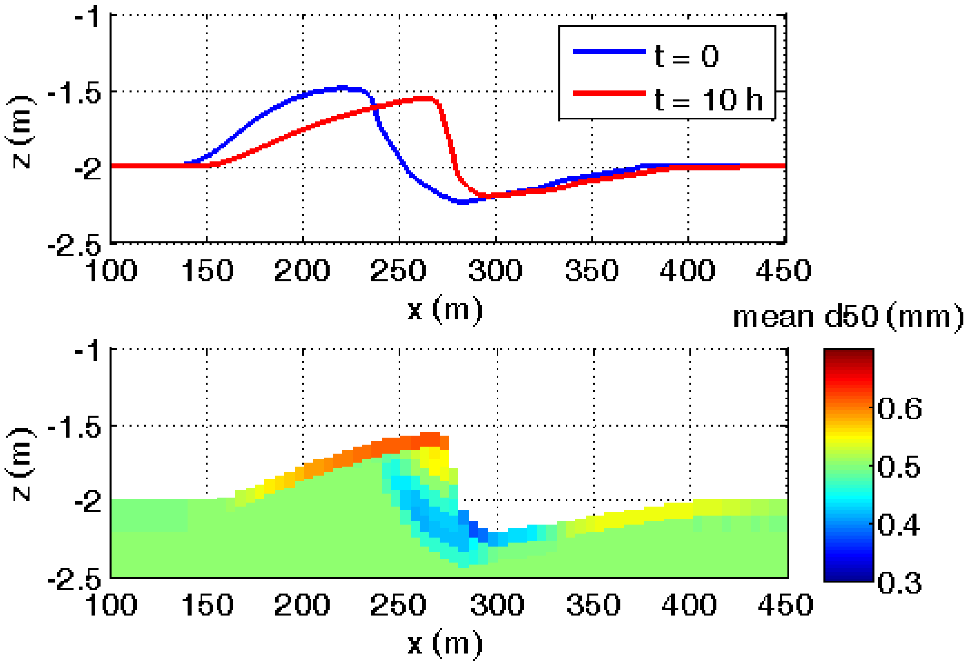
$$d50_{mean,L} = \prod_j \left(d50_{j,L}\right)^{F_{j,L}}$$

At each time step:

1) Sediment transport is computed and Exner equation is solved <u>for each class</u>,
2) Class fractions in each layer and transition layer thickness are updated,
3) New mean d50 is computed in each layer.



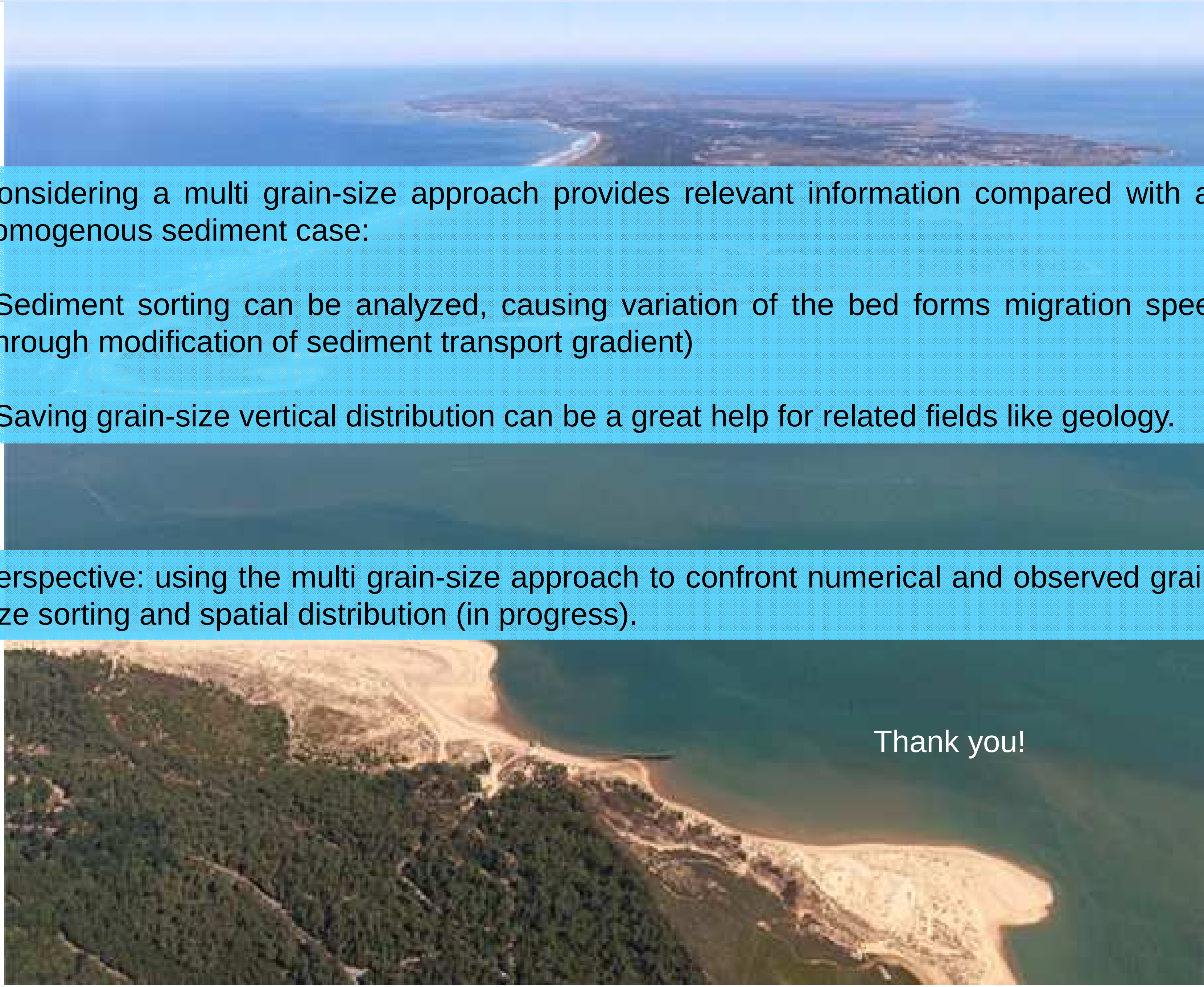
Euler central finite volume scheme



Euler WENO finite volume scheme

=> In response to the hydrodynamic forcing, coarser and finer sediment are respectively observed where erosion and deposition takes place (in agreement with *Holland and Elmore (2008)* for storm dominated Atlantic coast)

=> Adding WENO scheme to the multi-class multi-layer approach improves the stability, as in the homogeneous sediment case.



Considering a multi grain-size approach provides relevant information compared with an homogenous sediment case:

- Sediment sorting can be analyzed, causing variation of the bed forms migration speed (through modification of sediment transport gradient)

- Saving grain-size vertical distribution can be a great help for related fields like geology.

Perspective: using the multi grain-size approach to confront numerical and observed grain-size sorting and spatial distribution (in progress).

Thank you!